Recent Approaches in Knowledge Representation[1]

MOHAMMED SAEED, Ali Amer
University of Piteşti[2]
ali.amer81@gmail.com

Abstract

The main approaches dealing with imprecision modelling of knowledge in order to build Knowledge Representation and Processing Systems are considered in this paper. An overview on Zadeh's fuzzy models, Atanassov's intuitionistic fuzzy propositions, and Smarandache's neutrosophic extensions is given. These models can be applied to numbers, graphs, various maps, and expert systems.

Keywords: *fuzzy modelling, intuitionistic-fuzzy modelling, neutrosophic modelling, knowledge representation.*

ACM Classification: I.2.4.

1. *Introduction*

Any Knowledge Representation and Processing (KRP) system operating with imprecision data has to process input data coming from many sources: experts describing their knowledge on some field in natural language under subjectivity and ambiguity assumptions, sensors, instruments, approximate modelling. Not only incomplete data can be analysed before storing in KRP knowledge base, but also data inconsistency, uncertaint data, ambiguous data, fuzzy or vague data can be considered to be preprocessed before extracting knowledge.

Deal with imprecision during knowledge acquisition, representation and processing is one of the most challenging task when designing a KRP system. From numerical point of view, the following approaches were proposed by researchers starting from 1950: interval methods, fuzzy numbers, fuzzy intervals, intuitionistic fuzzy numbers and neutrosophic numbers. According to Moore (1966), in order to work with an uncertain number x, is better to work with an interval $[a, b]$ containing x. Also, given a function f, the value $f(x)$ will belong to an interval $[u, v]$ being also imprecise.

If $\diamond$ is an operator representing addition, subtraction, multiplication or division of real numbers, the corresponding operator in interval arithmetic

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[2]Doctoral School of Informatics.

interval will be denoted by $< \diamond >$, and the following rule is applied [16]: $[a, b] <\diamond> [c, d] = [\min(a \diamond c, a \diamond d, b \diamond c, b \diamond d), \max(a \diamond c, a \diamond d, b \diamond c, b \diamond d)]$, if and only if $x \diamond y$ is defined for $x \in [a, b]$, and $y \in [c, d]$. In the case of division, a rule taking into account the number zero is necessary: $1/[c, 0] = [-\infty, 1/c]$, and $1/[0, d] = [1/d, \infty]$. Combining numbers with intervals is easy because any number μ is the same as interval $[\mu, \mu]$.

Let X be a nonempty set representing the universe of discourse. A fuzzy set A of X is defined by all elements belonging to X according to a membership degree function f_A defined on X with values in $[0, 1]$. In this case, let us denote by A the following set: $A = (x, f_A(x)), x \in X$. This kind of sets were introduced by Zadeh (1965), who defined the non-membership degree by $1 - f_A(x)$, for every $x \in X$ [27, 29]. When someone changes the membership degree function then a new fuzzy set will be obtained.

If, for every $x \in X$, a non-membership degree function g_A is defined ($g_A : X \to [0, 1]$), with $f_A(x)+g_A(x)$ being less or equal 1), then the Atanassov intuitionistic fuzzy sets are introduced. In this case the value $1-f_A(x)-g_A(x)$ is called the degree of indeterminacy and expresses the lack of knowledge on x concerning its membership to the set A [2, 3, 4]. For fuzzy sets, there is no indeterminacy. A new extension to a three-valued representation was considered by Smarandache [21]. If $T_A(x)$ is the degree of membership, $F_A(x)$ is the degree of non-membership, and $I_A(x)$ is the degree of indeterminacy, the neutrosophic representation of a set A consists of three functions (T_A, I_A, F_A), such as $\forall x \in X$, $0 \leq T_A(x), F_A(x), I_A(x) \leq 1$, with ${}^{-}0 \leq T_A(x) + F_A(x) + I_A(x) \leq 3^{+}$, where for any real number ${}^{-}a$, and a^{+} are a sets of hyper-real numbers in non-standard analysis [18]: ${}^{-}a = \inf\{a-\epsilon, \epsilon \in R^*, \epsilon \text{ infinitesimal}\}$, and $a^{+} = \sup\{a + \epsilon, \epsilon \in R^*, \epsilon \text{ infinitesimal}\}$.

These approaches already inspired researchers to consider various types of graphs, knowledge maps, expert systems, and KRP systems which are described in this paper. The next section considers fuzzy KRP systems, the third section deals with intuitionistic fuzzy KRP systems, and the fourth section is dedicated to neutrosophic KRP systems.

2. *Fuzzy Modelling and Linguistic Knowledge Representation*

A fuzzy KRP system has rules such as *if A is true to a high degree then try to make B true to a high degree*, or *If x is large, y is medium, and z is low then t is good*, to mention some examples. Such KRP systems can be used as control systems (*if A is warm to some degree then B should be turned down to some value*) or is used to calculate the overall quality of a studied experiment (*if A is Large and B is Large then outcome is Very good, if C is Large and B is not Large then outcome is Poor*). More technical: If X_1 is A_1 and X_2 is A_2 and ... and X_n is A_n then Y_1 is B_1 and Y_2 is B_2 and ... and Y_m is B_m; X_i are variables whose values are fuzzy sets A_i; Y_j are variables whose values are fuzzy sets B_j where i and j follow some range. There is no loss of generality if we assume rules of the form If X is A and Y is B then Z is C.

The rules given as example make use of natural language expressions like "Large", "Very Good", "Poor" etc. Such items are called values for some linguistic variables. According to Zadeh (1975), a linguistic variable has a *name*, like Age, a colection of values called the *term-set*, a syntactic rule to generate the elements of the *term-set*, a universe of discourse U, and a semantic rule M such as if X is a linguistic variable, $M(X)$ is a fuzzy set of U. The compatibility of *Age* 27 with *young* is given by a "membership" degree, for instance 0.7.

In the context of expert systems in medicine using fuzzy concepts, the following description is a complete example: the linguistic variable X = "blood glucose level", the term-set T = {'slightly increased', 'increased', 'significantly increased', 'strongly increased'}, the universe of discourse is U = [100, 600], M(X) is given by trapezoidal membership functions, for $X \in T$, M(X)(u) is the membership degree for "u is X". The formal grammar generating T is very simple.

In order to provide answers to querries formulated by any end-user, the inferential engine of the KRP system should be able to apply the most appropriate fuzzy implication procedure. Following Atanassov et al. (2007), an implication operator $I(x, y)$ should listen to some axioms from the following set: A1) $(\forall x)(\forall y)(x \leq y \rightarrow (\forall z)(I(x, z) \geq I(y, z))$; A2) $(\forall x)(\forall y)(x \leq y \rightarrow (\forall z)(I(z, x) \geq I(z, y))$; A3) $(\forall y)(I(0, y) = 1)$; A4) $(\forall y)(I(1, y) = y)$; A5) $(\forall x)(I(x, x) = 1)$; A6) $(\forall x)(\forall y)(\forall z)(I(x, I(y, z)) = I(y, I(x, z)))$; A7) $(\forall x)(\forall y)(I(x, y) = 1$ if and only if $x \leq y)$; A8) $(\forall x)(\forall y)(I(x, y) = I(n(y), n(x)))$, with n a negation operator; A9) I is a continuous function. All axioms are verified by the Lukasiewicz's implication given by $I(x, y) = \min(1, 1 - x + y)$. Only the first seven axioms are verified by the Godel's implication: $I(x, y) = 1$ if $x \leq y$, and $I(x, y) = y$, if $x > y$.

Therefore, one Fuzzy KRP system is composed by: Input module (crisp data), Fuzzification Unit, Fuzzy Knowledge Base, Fuzzy Inference Engine, Defuzzification Unit, Output module (crisp).

3. ***Intuitionistic Fuzzy Knowledge Representation***

Let X be the nonempty set of objects stored in a KPR system. In an intuitionistic fuzzy framework A, for every x will be defined both $f_A(x)$ and $g_A(x)$, representing the *degree of membership*, and the *degree of non-membership*, respectively. Other characteristics are possibly to be introduced: the *degree of indeterminacy* $h_A(x) = 1 - f_A(x) - g_A(x)$, the *degree of favour* of x like $m_A(x) = f_A(x)(1 + h_A(x))$, and the *degree of against* of x like $n_A(x) = g_A(x)(1 + h_A(x))$.

Two intuitionistic fuzzy sets of X, denoted by A, and B, are similarly if there is $x \in X$ such that $f_A(x) = f_B(x)$, and $g_A(x) = g_B(x)$. If these properties apply for all $x \in X$, then A and B are equal, or comparable. Also, two intuitionistic fuzzy sets A and B are equivalent if and only if a bijection h is identified in order to have: $f_B(x) = h(f_A(x))$, and : $g_B(x) = h(g_A(x))$, for every $x \in X$. Hence, $f_A(x) = h^{-1}(f_B(x))$ and $g_A(x) = h^{-1}(g_B(x))$.

The concepts of "subset", respectively "proper subset" is defined by the direct order relation between membership degrees and the inverse order relation between non-membership degrees, namely: $A \subseteq B$ if and only if $f_A(x) \leq f_B(x)$, and $g_A(x) \geq g_B(x)$, and $A \subset B$ if and only if $f_A(x) < f_B(x)$, and $g_A(x) > g_B(x)$, respectively.

Let x be a proposition modelled by intuitionistic fuzzy logic. Hence, its truth-value is given by $V(x) =< a, b >$, where a and b are the degrees of validity and of non-validity of x, using the notations of Atanassov et al. (2007). The intuitionistic fuzzy proposition x is an Intuitionistic Fuzzy Tautology (IFT) if and only if $a \geq b$. Let x and y be two intuitionistic fuzzy propositions, given by $V(x) =< a, b >$, and $V(y) =< c, d >$. Let $I(a, b; c, d)$ be the intuitionistic fuzzy implication (IFI) operator. With the axioms introduced in previous section, the Zadeh's IFI given by $I(a, b; c, d) =< \max(b, \min(a, c)), \min(a, d) >$ satisfies the axioms A2/3/4/5/7/9, while the Kleene-Dienes's IFI given by $I(a, b; c, d) =< \max(b, c), \min(a, d) >$ satisfies all axioms except A7.

The IFS-based Composition Rule of Inference (ICLRI), according to Cornelis & Deschrijver (2001), makes use of two universes of discourse U and V, two variables X and Y assuming values in U, respective V, and an intuitionistic fuzzy relation R between U and V. ICLRI can be expressed as an inference scheme by {X is A', (X,Y) is R; Y is B' = R ∘ A'}, where A' is an intuitionistic fuzzy set of U. ICLR defines the Intuitionistic Generalized Modus Ponens (IGMP) scheme. It is important, when building an IFS-based KRP, to use only implication operators which assure the validity of the modus ponens.

Both fuzzy numbers and intuitionistic fuzzy numbers can be used in the context of KRP systems supporting imprecision. The most used categories are represented by triangular, respective trapezoidal intuitionistic fuzzy numbers. In some way, intuitionistic fuzzy representation is a particular case of neutrosophic representation with the sum of all three components equal to unity.

Hence, one Intuitionistic Fuzzy KRP system is composed by: Input module (crisp data), Intuitionistic Fuzzification Unit, Intuitionistic Fuzzy Knowledge Base, Intuitionistic Fuzzy Inference Engine, Intuitionistic Defuzzification Unit, Output module (crisp).

4. ***Neutrosophic models in Knowledge Representation***

The neutrosophic set, from philosophical point of view, is modelled by values from the real standard or non-standard subsets of $]^-0, 1^+[$. Since the non-standard interval $]^-0, 1^+[$ is difficult to be used for real engineering and scientific problems, the standard unit interval $[0, 1]$ is used. An element x of the univers of discourse is called significant with respect to neutrosophic set A of X if the following degrees are significant: the degree of truth-membership or falsity-membership or indeterminancy-membership value, i.e., $T_A(x)$ or $I_A(x)$ or $F_A(x)$ is greater than 0.5. An intuitionistic neutrosophic set is defined by $\tilde{A} =< x, T_A(x), I_A(x), F_A(x) >$, where $\min\{T_A(x), F_A(x)\} \leq 0.5, \min\{T_A(x), I_A(x)\} \leq 0.5$, and $\min\{F_A(x), I_A(x)\} \leq 0.5$, $\forall x \in X$, when $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 2$.

Neutrosophic logic operators can be defined in many ways depending on the needs of applications or the problem to be solved. N-norms (N_n) and N-conorms (N_c) can be used. N_n represents the *and* operator in neutrosophic logic and the *intersection* operator in neutrosophic set theory. N_c represents the *or* operator in neutrosophic logic and the *union* operator in neutrosophic set theory. Both N_n and N_c should satisfy the following axioms: a) *Boundary Conditions for* N_n: $N_n(x, 0) = 0$, $N_n(x, 1) = x$; b) *Boundary Conditions for* N_c: $N_c(x, 1) = 1$, $N_c(x, 0) = x$; c) *Commutativity*: $N(x, y) = N(y, x)$; d) *Monotonicity*: if $x \leq y$ then $N(x, z) \leq N(y, z)$; e) *Associativity*: $N(N(x, y), z) = N(x, N(y, z))$, where $N \in \{N_n, N_c\}$.

A general example of N-norm is built as $N_n(x, y) = (T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2)$, where $x(T_1, I_1, F_1)$, $y(T_2, I_2, F_2)$, $\wedge$ is a N-norm, and $\vee$ is any N-conorm. Similarly, a general example of N-conorm is built as $N_c(x, y) = (T_1 \vee T_2, I_1 \wedge I_2, F_1 \wedge F_2)$. For instance $a \wedge b = ab$ (as product), and $a \vee b = a + b - ab$. A negation operator can be easy defined by $n(x) = y$, where $x(T, I, F)$, and $y(F, I, T)$. An implication operator can be introduced by $I(x, y) = N_c(n(x), y)$. However, depending on the constituents used to define one N-norme (resp. N-conorme), many definitions for the implication operator can be obtained.

Representing knowledge by neutrosophic numbers presumes an indeterminancy component to be added. Formally, a neutrosophic number is $a + b\mathrm{I}$, where a and b are real or complex numbers, and I is the indeterminacy operator with $I^2 = I$. The concept can be extended to neutrosophic matrices and to neutrosophic algebraic structure. According to our aim, a more interesting extension is represented by neutrosophic graphs, with indeterminacy at vertex level, or at edge level. The indeterminacy component (vertex or edge) is used when incomplete information exists about such entities. The adjacency matrix of a neutrosophic graph consists of elements belonging to the set {0 - no connection, 1 - connection, I - indeterminate connection between vertices}.

The neutrosophic cognitive maps and neutrosophic relational maps are generalizations of fuzzy cognitive maps and respectively fuzzy relational maps. A Neutrosophic Cognitive Map represents the causal relationship between concepts, the neutrosophic adjacency. The corresponding neutrosophic adjacency matrix related to a neutrosophic cognitive map has values belonging to the set {0, 1, -1, I}, where 0, 1 and I are as above, and -1 describes an inverse proportionaly connection.

Therefore, one Neutrosophic KRP system is composed by: Input module (crisp data), Neutrosophication Unit, Neutrosophic Knowledge Base, Neutrosophic Inference Engine, Deneutrosofication Unit, Output module (crisp). The neutrosophication unit preprocesses crisp input data to identify valid cases, invalid cases, ambiguous cases. Every item in the knowledge base is described by three components to be used for inferential operations on the content. Once identified neutrosophic operators for logical connectors, the neutrosophic rule base can be used as in classical framework of expert systems implemented by logic programming or production rules. The Deneutrosophication Unit is responsible with filtering membership/validity information in order to provide a centre of gravity, or a particular mean of data.

5. *Conclusions*

This paper had presented some approaches on knowledge representation when incomplete, ambiguous or imprecise data sets are collected. An overview on Zadeh's fuzzy models, Atanassov's intuitionistic fuzzy propositions, and Smarandache's neutrosophic extensions is given. These models can be applied to numbers, graphs, various maps, and KRP systems.

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