Neutrosophic Computational Models – II

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Abstract
This paper continues to describe neutrosophic computational models based on distances and similarity measures. Both single value and interval neutrosophic sets are considered. The usage of a software library in implementing future projects is proposed.
Keywords: *neutrosophic sets, single valued neutrosophic sets, interval neutrosofic sets, Hausdorff-Pompeiu distance, similarity measures*
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1. **Introduction**

Smarandache, starting from 1995, defined the neutrosophy [12] and introduced neutrosophic sets as a generalization of Zadeh's fuzzy sets [19] and Atanassov's intuitionistic fuzzy sets [2]. It was his idea to use the notations t, i, and Ff to denote the truth-membership, indeterminacy-membership, and the falsity-membership degrees as real standard or non-standard subsets of $]0^-, 1^+[$, for every element x of a crisp set X of points (object) in order to define a neutrosophic set A in X: $t_A(x) : x \to]0^-, 1^+[$, $i_A(x) : x \to]0^-, 1^+[$, and $f_A(x) : x \to]0^-, 1^+[$. For basic properties of operations on general neutrosophic sets, see the survey [1]. In the following, only two particular cases of neutrosophic sets will be considered: single value neutrosophic sets, and interval neutrosophic sets over finite universe X.

According to [17], a single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function t_A, indeterminacy-membership function i_A, and falsity-membership function f_A, such as for each point $x \in X$, $t_A(x), i_A(x), f_A(x) \in [0, 1]$. Wang et al [16], presented the fundamental properties of interval neutrosophic sets (INS) A when $t_A(x), i_A(x), f_A(x) \subset [0, 1]$, for every $x \in X$.

The complement of a SVNS A, denoted by $c(A)$ is immediately defined as $t_{c(A)} = f_A$, $i_{c(A)} = 1 - i_A$, and $f_{c(A)} = t_A$, but in the case when A is INS, the definition is based on: $t_{c(A)} = f_A$, $\inf i_{c(A)} = 1 - \sup i_A$, $\sup i_{c(A)} = 1 - \inf i_A$, and $f_{c(A)} = t_A$.

If A and B are two SVNSs in X then $A \subset B$ if and only if, for all $x \in X$, $t_A(x) \leq t_B(x)$, $i_A(x) \geq i_B(x)$, and $f_A(x) \geq f_B(x)$. Therefore, $A = B$ if and only if $t_A(x) = t_B(x)$, $i_A(x) = i_B(x)$, and $f_A(x) = f_B(x)$, for all $x \in X$.

Also, If A and B are two INSs in X then $A \subset B$ if and only if for every $x \in X$, $\inf t_A(x) \leq \inf t_B(x)$, $\sup t_A(x) \leq \sup t_B(x)$, $\inf i_A(x) \geq \inf i_B(x)$, $\sup i_A(x) \geq \sup i_B(x)$, $\inf f_A(x) \geq \inf f_B(x)$, and $\sup f_A(x) \geq \sup f_B(x)$. Therefore, $A = B$ if and only if $\inf t_A(x) = \inf t_B(x)$, $\sup t_A(x) = \sup t_B(x)$, $\inf i_A(x)$=$\inf i_B(x)$, $\sup i_A(x)$=$\sup i_B(x)$, $\inf f_A(x)$=$\inf f_B(x)$, and $\sup f_A(x) = \sup f_B(x)$, for every $x \in X$.

The 'similarity' of two neutrosophic sets (SVNS or INS) can be defined by various measures, including those based on distances. This paper outlines, in the next section, various approaches in distance computation between neutrosophic sets. Both single value and interval neutrosophic sets are considered. In the third section, there are introduced similarity measures, and in the last section, further remarks are given on the usage of a software library in implementing future projects.

2. **Distance between neutrosophic sets**

Let $N(X)$ be the neutrosophic sets over X, and d be a real function $d : N(X) \times N(X) \to R^+$. d is called a distance measure if it satisfies the following conditions: (d_1) $d(A,B) = d(B,A)$, for all $A, B \in N(X)$; (d_2) $d(A,A) = 0$, for all $A \in N(X)$; (d_3) For every $(A,B,C) \in N(X) \times N(X) \times N(X)$ such as $A \subset B \subset C$, then $d(A,B) \leq d(A,C)$ and $d(B,C) \leq d(A,C)$.

Science, based on constraints imposed by applications, asks for normalized distances. In the following both standard and normalized distances are considered when the universe X is finite, and $X = \{x_1, x_2, \cdots, x_n\}$, with n elements.

Let us assume that A and B are any SVNS, and for every $i, i \in \{1, 2, \cdots, n\}$, there are available the degrees $(t_A(x_i), i_A(x_i), f_A(x_i))$, and $(t_B(x_i),\ i_B(x_i), f_B(x_i))$ as values in $[0,1]$.

The Hamming distance between A and B is defined by [9]:

$$d_1(A,B) = \sum_{i=1}^{n} \{|t_A(x_i) - t_B(x_i)| + |i_A(x_i) - i_B(x_i)| + |f_A(x_i) - f_B(x_i)|\}, \quad (1)$$

and $0 \leq d_1(A,B) \leq 3n$. The normalized Hamming distance between A and B is obtained as

$$n_1(A,B) = \frac{1}{3n} d_1(A,B), \quad (2)$$

with $0 \leq n_1(A,B) \leq 1$.

The Euclidean distance between A and B is defined by [9]:

$$d_2(A,B) = \left\{ \sum_{i=1}^{n} \{|t_A(x_i) - t_B(x_i)|^2 + |i_A(x_i) - i_B(x_i)|^2 + |f_A(x_i) - f_B(x_i)|^2\} \right\}^{\frac{1}{2}}, \quad (3)$$

and $0 \leq d_2(A,B) \leq \sqrt{3n}$. The normalized Euclidean distance between A and B is obtained as

$$n_2(A,B) = \frac{1}{\sqrt{3n}} d_2(A,B), \tag{4}$$

with $0 \leq n_2(A,B) \leq 1$.

In general, the p-distance between A and B can be defined as

$$d_p(A,B) = \left\{ \sum_{i=1}^{n} \{|t_A(x_i) - t_B(x_i)|^p + |i_A(x_i) - i_B(x_i)|^p + |f_A(x_i) - f_B(x_i)|^p\} \right\}^{\frac{1}{p}}, \tag{5}$$

and $0 \leq d_p(A,B) \leq (3n)^{1/p}$. The normalized p-distance between A and B is obtained by

$$n_p(A,B) = \frac{1}{(3n)^{1/p}} d_p(A,B), \tag{6}$$

with $0 \leq n_p(A,B) \leq 1$.

The weighted distance between A and B, for a sequence of positive numbers $0 \leq w_i \leq 1, i \in \{1,2,\cdots,n\}$, and $\sum_{i=1}^{n} w_i = 1$, can be defined by

$$d_w(A,B) = \left\{ \frac{1}{3} \sum_{i=1}^{n} w_i \{|t_A(x_i) - t_B(x_i)|^p + |i_A(x_i) - i_B(x_i)|^p + |f_A(x_i) - f_B(x_i)|^p\} \right\}^{\frac{1}{p}}. \tag{7}$$

A Hausdorff-Pompeiu type distance between A and B can be introduced in a manner similar those considered by Szmidt and Kaprzyk for intuitionistic-fuzzy sets [15], and Broumi and Smarandache for neutrosophic sets [6]:

$$d_H(A,B) = \frac{1}{n} \sum_{i=1}^{n} \max\{|t_A(x_i) - t_B(x_i)|, |i_A(x_i) - i_B(x_i)|, |f_A(x_i) - f_B(x_i)|\}, \tag{8}$$

with $0 \leq d_H(A,B) \leq 1$.

The above formulas can be extended for interval neutrosophic sets (INSs) A and B, where for every $i, i \in \{1,2,\cdots,n\}$, there are available the degrees $(t_A(x_i), i_A(x_i), f_A(x_i))$, and $(t_B(x_i), i_B(x_i), f_B(x_i))$ as subintervals of $[0,1]$.

The Hamming distance between the INSs A and B is defined by [18]:

$$\begin{aligned} \delta_1(A,B) = \; & \tfrac{1}{6} \textstyle\sum_{i=1}^{n} \{|\inf t_A(x_i) - \inf t_B(x_i)| + |\sup t_A(x_i) - \sup t_B(x_i)| + \\ + \; & |\inf i_A(x_i) - \inf i_B(x_i)| + |\sup i_A(x_i) - \sup i_B(x_i)| + \\ + \; & |\inf f_A(x_i) - \inf f_B(x_i)| + |\sup f_A(x_i) - \sup f_B(x_i)|\}, \end{aligned} \tag{9}$$

while the normalized Hamming distance becomes:

$$\begin{aligned} \nu_1(A,B) = \; & \tfrac{1}{6n} \textstyle\sum_{i=1}^{n} \{|\inf t_A(x_i) - \inf t_B(x_i)| + |\sup t_A(x_i) - \sup t_B(x_i)| + \\ + \; & |\inf i_A(x_i) - \inf i_B(x_i)| + |\sup i_A(x_i) - \sup i_B(x_i)| + \\ + \; & |\inf f_A(x_i) - \inf f_B(x_i)| + |\sup f_A(x_i) - \sup f_B(x_i)|\}. \end{aligned} \tag{10}$$

The Euclidean distance between the INSs A and B is defined by:

$$\begin{aligned} \delta_2(A,B) = \; & \Big\{ \tfrac{1}{6} \textstyle\sum_{i=1}^{n} \{|\inf t_A(x_i) - \inf t_B(x_i)|^2 + |\sup t_A(x_i) - \sup t_B(x_i)|^2 + \\ + \; & |\inf i_A(x_i) - \inf i_B(x_i)|^2 + |\sup i_A(x_i) - \sup i_B(x_i)|^2 + \\ + \; & |\inf f_A(x_i) - \inf f_B(x_i)|^2 + |\sup f_A(x_i) - \sup f_B(x_i)|^2\} \Big\}^{\frac{1}{2}}, \end{aligned} \tag{11}$$

and the normalized Euclidean distance between INSs A and B is obtained as

$$\nu_2(A,B)= \Big\{\tfrac{1}{6n}\sum_{i=1}^{n}\{|\inf t_A(x_i)–\inf t_B(x_i)|^2+|\sup t_A(x_i)–\sup t_B(x_i)|^2+ \\ + |\inf i_A(x_i)–\inf i_B(x_i)|^2+|\sup i_A(x_i)–\sup i_B(x_i)|^2+ \\ + |\inf f_A(x_i)–\inf f_B(x_i)|^2+|\sup f_A(x_i)–\sup f_B(x_i)|^2\}\Big\}^{\frac{1}{2}}. \quad (12)$$

Similarly, the p-distance between INSs A and B is:

$$\delta_p(A,B)= \Big\{\tfrac{1}{6}\sum_{i=1}^{n}\{|\inf t_A(x_i)–\inf t_B(x_i)|^p+|\sup t_A(x_i)–\sup t_B(x_i)|^p+ \\ + |\inf i_A(x_i)–\inf i_B(x_i)|^p+|\sup i_A(x_i)–\sup i_B(x_i)|^p+ \\ + |\inf f_A(x_i)–\inf f_B(x_i)|^p+|\sup f_A(x_i)–\sup f_B(x_i)|^p\}\Big\}^{\frac{1}{p}}, \quad (13)$$

while the normalized p-distance between INSs A and B is obtained as

$$\nu_p(A,B)= \Big\{\tfrac{1}{6n}\sum_{i=1}^{n}\{|\inf t_A(x_i)–\inf t_B(x_i)|^p+|\sup t_A(x_i)–\sup t_B(x_i)|^p+ \\ + |\inf i_A(x_i)–\inf i_B(x_i)|^p+|\sup i_A(x_i)–\sup i_B(x_i)|^p+ \\ + |\inf f_A(x_i)–\inf f_B(x_i)|^p+|\sup f_A(x_i)–\sup f_B(x_i)|^p\}\Big\}^{\frac{1}{p}}. \quad (14)$$

When considering the weighted p-distance between INSs A and B, the following relation is obtained:

$$\delta_{wp}(A,B)= \Big\{\tfrac{1}{6}\sum_{i=1}^{n} w_i\{|\inf t_A(x_i)–\inf t_B(x_i)|^p+|\sup t_A(x_i)–\sup t_B(x_i)|^p+ \\ + |\inf i_A(x_i)–\inf i_B(x_i)|^p+|\sup i_A(x_i)–\sup i_B(x_i)|^p+ \\ + |\inf f_A(x_i)–\inf f_B(x_i)|^p+|\sup f_A(x_i)–\sup f_B(x_i)|^p\}\Big\}^{\frac{1}{p}}, \quad (15)$$

where $0 \le w_i \le 1, i \in \{1, 2, \cdots, n\}$, and $\sum_{i=1}^{n} w_i = 1$.

A Hausdorff-Pompeiu type distance between INSs A and B can be given by:

$$\delta_H(A,B)= \tfrac{1}{2n}\sum_{i=1}^{n}\max\{|\inf t_A(x_i)–\inf t_B(x_i)|+|\sup t_A(x_i)–\sup t_B(x_i)|, \\ |\inf i_A(x_i)–\inf i_B(x_i)|+|\sup i_A(x_i)–\sup i_B(x_i)|, \\ |\inf f_A(x_i)–\inf f_B(x_i)|+|\sup f_A(x_i)–\sup f_B(x_i)|\}, \quad (16)$$

with $0 \le \delta_H(A,B) \le 1$.

3. **Distance based similarity measures**

Let s be a real function $s : N(X) \times N(X) \to R^+$. The function s is a similarity measure if it satisfies the following conditions: (s_1) $s(A,B) = s(B,A)$, for all $A, B \in N(X)$; (s_2) $s(A,A) = 1$, for all $A \in N(X)$; (s_3) For every $(A,B,C) \in N(X) \times N(X) \times N(X)$ such as $A \subset B \subset C$, then $s(A,B) \ge d(A,C)$ and $s(B,C) \ge d(A,C)$.

Let d be any normalized distance presented above. Firstly, let s_d^1 be defined such as

$$s_d^1(A,B) = 1 - d(A,B). \quad (17)$$

Obviously, $0 \le s_d^1(A,B) \le 1$.

If d is a non-normalized distance, let s_d^2 be defined such as

$$s_d^2(A,B) = \frac{1}{1 + d(A,B)}. \quad (18)$$

This definition can be used also for normalized distances, but the minimum value of the similarity measure is 0.5.

Let, also, be s_d^3 defined by

$$s_d^3(A,B) = \exp^{-\alpha d(A,B)}, \tag{19}$$

where α is a positive parameter, to be established experimentally, depending on the real-life application.

Based on the distance's properties, it is easy to prove that s_d^1, s_d^2, and s_d^3 are similarity measures.

4. **Applications**

Similarity measures are useful to solve problems in bioinformatics, computer vision, information retrieval, data classification, outliers identification, decision making, and many other fields. To exemplify, let us consider the case of multiple attribute decision making (MADM) which is a well-known subfield of decision making. Various algorithms were proposed to solve MADM problems. Some tracks are possible during research: i) If attributes are modelled by neutrosophic characteristics then similar alternatives/objects can be identified using distance matrix or similarity matrix; ii) If there exists an ideal solution/object then alternatives having the highest similarity to the ideal solution/object will be selected.

Ranking items (objects, networks, alternatives etc.) described by neutrosophic-modelled features (attributes) consists of assigning to each item a ranking score. The ranking score can be obtained by computing distance/similarity measure from/with the ideal item. If the items are experts evaluating a process or a state of some system when multiple features are considered then a consensus ranking based on similarity measures is possible.

5. **Concluding remarks and future projects**

This paper described various models for distances and similarity measures applied to neutrosophic entities. To solve real-life problems modelled by soft computing concepts (fuzzy, intuitionistic-fuzzy, neutrosophic), an Application Programming Interface supporting the computation of distance and similarity measure for both classes (SVNSs and INSs) of neutrosophic sets was designed and preliminary results on multiple attribute decision making are encouragement. A future project is dedicated to consensus ranking based on neutrosophic similarity measures for the evaluation of multimedia objects.

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