ON THE SELF-ORGANIZING PATTERN WITH APPLICATION TO SMART MATERIALS

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Abstract

The smart materials are organizing in space and time, forming a variety of complex structures, during the self-organization pattern. In this paper, we apply the Turing model in order to obtain highly structured forms from a single perturbation.

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ACM/AMS Classification: 34D10

1. Introduction

The self-organizing phenomenon is found in many disciplines, starting from the natural sciences such as physics, complexity science, biology, mathematics, and engineering of smart materials and finishing with social sciences such as economics and anthropology. Self-organization has also been observed in mathematical systems such as cellular automata. A cellular automaton is a discrete model consisted of a regular grid of cells, each in one of a finite number of states (Wolfram 1983). This phenomenon consists by a spontaneous transition from chaos to order in the most important properties of materials, especially the smart or intelligent materials (Maselko 1996).

The self-organizing phenomena are related to the phenomenological universality which represents a useful tool for the classification and interpretation of different nonlinear phenomena in the context of cross-disciplinary research in nature and society. At the micron-scale, the properties of materials depend on size and the size depends on the external effects. Also, at the nano-scale, the properties of matreials depend on size and agglomeration, and size and agglomeration depend on external effects. Therefore, scaling is a very powerful tool for the quest of universal laws in all sciences. In fact, the description of

life processes in micro- and macro population dynamics, cover more than 27 orders of magnitude (in volume) from beneficial insects and animals, especially rare species to common insects and animals (Munteanu, Delsanto and Gliozzi 2010). Scaling laws are interesting instruments to analyze the manifestations of intrinsic mechanisms (such as energy conservation, phase transitions, complexity and/or randomness), which may be basically the same even in very different fields (Delsanto et al. 2008; West, Brown and Enquist 1997).

The main chaos characteristic is the impossibility of making predictions for long-term intervals for the systems evolution based on the finite precision measurement (Guckenheimer and Holmes 1983). Despite the complexities of chaotic behavior, the same main characteristic can be intelligently exploited to direct the system to some desired state, using a chosen sequence of small perturbations to some system parameter. This approach, which is of fundamental interest for the control system, is called targeting (Shinbrot et al 1992). Control of chaos is also making substantial contribution in the field of mechanics (Stanescu et al 2007, 2011) and astrodynamics, especially related to the exciting issue of low-energy transfer (Macau and Grebogi 2006). The control of chaos is making substantial contribution in the field of astrodynamics, particularly related to the important and timely issue of low-energy transfer (Macau 2000, 2003; Macau and Caldas 2008).

This paper is devoted to the describing of the self-organizing Turing pattern applied to the smart materials, i.e. materials which can change their properties in a predictable or controllable way due to the phase transition, as response to the external loading. An attempt to understand such materials from a physico-chemical perspective is the Turing stationary spatial structures, based on the Belousov-Zhabotinski (BZ) reaction. Turing has introduced in 1952 a reaction-diffusion system for two chemical substances called morphogens, and developed shapes and structures found in nature.

2. **The Turing pattern formation**

Nature abounds with examples of beautiful patterns, like flowers or sea-shell shapes (Fig.1). The spatial shapes are developed by the environment actions (the cases of thin shells discussed by Mihailescu and Chiroiu 2004).

As concerning the smart materials field, the interesting properties of the piezoelectric materials, dielectric elastomers, shape memory alloys, liquid crystals or ferrofluids sugest the using of the Turing pattern which is based on the propagating of chemical waves of the BZ type, combined with the effects of diffusion. It is known that the diffusion can create inhomogeneities, and if the conditions are just right then the inhomogeneities will lead to a stationary spatial structure, called a Turing structure.

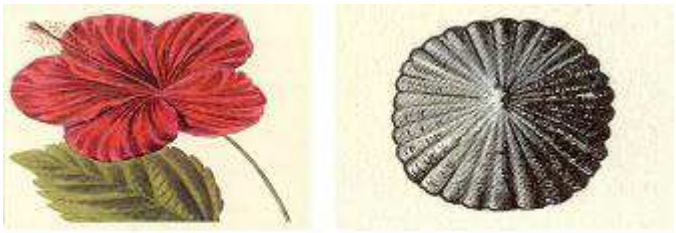

Fig. 1. Nature undevelopable surfaces (Mihailescu and Chiroiu 2004).

For the structures that do not have an equilibrium state, the particular patterns that are formed depend on the initial conditions. This means one can obtain a great deal of variety in one simulation. Furthermore, the patterns also depend on the geometry of the surface. The Turing system is represented by two coupled reaction-diffusion equations (Leppänen et al. 2004),

$$\begin{aligned} u_t &= D_u \nabla^2 u + f(u,v), \\ v_t &= D_v \nabla^2 v + g(u,v) \end{aligned} \tag{1}$$

where $u(x,t)$ and $v(x,t)$ are the concentrations of substances, and D_u, D_v are the time scaled diffusion coefficients. The functions f and g describe the reaction kinetics, as follows

$$\begin{aligned} f(u,v) &= \alpha_1 u + \alpha_2 v + \alpha_3 uv + \alpha_4 uv^2 + \alpha_5 vu^2, \\ g(u,v) &= \beta_1 u + \beta_2 v + \beta_3 uv + \beta_4 uv^2 + \beta_5 vu^2 \end{aligned} \tag{2}$$

where parameters α_i, β_i, $i = 1, ...5$ are the amplitudes of various terms, alloowing the control for favouring a certain pattern formation.

A particular form of (1) and (2) is the Belousov-Zhabotinski (ZB) reaction, which is a chemical reaction that maintains the non-equilibrium state of a structure for a long time, leading to macroscopic temporal oscillations and spatial pattern formation. This behaviour is called a chemical clock, and is often studied as a prototype of a simple biological system. The dimensionless form of ZB equations are

$$\begin{aligned} \varepsilon x_\tau &= qy - xy + x(1-x), \\ \delta y_\tau &= -qy - xy + fz, \\ z_\tau &= x - z \end{aligned} \tag{3}$$

The stability of the pattern formation is controlled by using the Maxau and Grebogi method to stabilize the chaos (Macau and Grebogi 2006). The basic idea of controlling the chaos consists into using the unstable periodic orbits. The idea is based on the fact that the unstable periodic orbits are embedded in the chaotic behavior set according to some performance criteria, and consequently, it is possible to define a small region around the desired

periodic orbit. The chaotic trajectory is transitive in its invariant set, and in consequence, by starting from any initial condition, the trajectory will transfer to the small region about the chosen unstable periodic orbits, after some time. When this occurs, small chosen parameter perturbations are applied to force and retain the trajectory evolving about unstable periodic orbits.

A short describing of the Macau and Grebogi method is given next, for a discrete time-dynamical system

$$X_{i+1} = F\left(X_i, p\right) \tag{4}$$

where $X_i \in \mathbb{R}^2$, and $p \in \mathbb{R}$ is an externally controllable parameter and F a smooth vector function in both variables. The nominal value of p for which F is chaotic on a compact, invariant set $\Lambda \subset \mathbb{R}^n$, is noted by $p = \bar{p}$. The parameter perturbation used to control the system is required to be small

$$|p - p_0| < \delta \tag{5}$$

where p_0 is the nominal parameter and δ a small number. The goal is to choose p in such a way that a typical trajectory in the chaotic region is stabilized about the desirable unstable periodic orbit. The stabilization procedure starts only when the chaotic trajectory enters a small region around one of the periodic orbit points, whose size is proportional to δ. Once the particle is inside this small region, p is changed to keep the trajectory near the unstable periodic orbit.

Let us assume that the unstable periodic orbit of period n is

$$X_{01}\left(p\right) \to X_{02}\left(p\right) \to ... \to X_{0n}\left(p\right) \to X_{0(n+1)}\left(p\right) = X_{01}\left(p\right). \tag{6}$$

The linearized dynamics in the neighborhood of the period-n orbit is

$$X_{n+1} - X_{0(n+1)}\left(p_n\right) = M\left[X_n - X_{0n}\left(p_n\right)\right], \tag{7}$$

where M is 2D Jacobian matrix at the orbit point X_{0n}, expressed in terms of its stable and unstable directions, and $p_n = p_0 + \left(\Delta p\right)_n g_n$, where $\left(\Delta p\right)_n < \delta$ from (7). The parameter variation is given by

$$X_{0n}\left(p_n\right) - X_{0n}\left(p_0\right) \approx \left(\Delta p\right)_n g_n, \tag{8}$$

where $g_n = \frac{\partial X_{0n}(p_n)}{\partial p}\,|_{p_0}$. When M has complex-conjugate eigenvalues, the stable and unstable directions can be determined. To find the stable direction at X_0, we iterate the point N times in the forward direction under the map F and get the trajectory

$$X_1 = F\left(X_0\right), X_2 = F\left(X_1\right) = F^2\left(X_0\right), X_N = F\left(X_{N-1}\right) = F^{(N)}\left(X_0\right).$$

Next we consider at X_N, a circle of small radius ε. If this circle is iterated backward once, it will become an ellipse at X_{N-1} with the major axis along the stable direction at X_{N-1}. This ellipse is iterated backward, while its major

axis is kept of the order of ε via certain normalization method. This procedure is repeated all the way back to X_0, where the ellipse becomes very thin, with its major axis along the stable direction, provided N is large enough. To control the orbit, it is required that the next iteration of a trajectory point, after falling into one of the small neighborhoods around X_{0n}, lies on the stable direction at $X_{0(n+1)}(p_0)$

$$\left[X_{n+1} - X_{0(n+1)}(p_0)\right] \cdot f_{u(n+1)} = 0. \tag{9}$$

With M evaluated at $X_{0n}(p_0)$, by substituting equations (7) and (8) into (9), we obtain the following expression for the parameter perturbations

$$(\Delta p)_n = \frac{\left\{M\left[X_{n+1} - X_{0(n+1)}(p_0)\right]\right\} \cdot f_{u(n+1)}}{\left[(Mg_n) - g_{n+1}\right] \cdot f_{u(n+1)}}. \tag{10}$$

The parameter perturbation (10) is applied at each time-step of the trajectory, which is kept stabilized around the unstable period orbit.

Let us return to (4) and consider a ball $B_\varepsilon(X_t)$ of radius ε. The targeting goal is to find a constructive orbit that goes from a point $px_s \in B_\varepsilon(X_s)$ to $px_t \in B_\varepsilon(X_t)$. By using this constructive orbit, the chaos is intelligently exploited to direct trajectories to a desired state in the shortest possible time, by using a chosen sequence of small perturbation to some control parameters. Since these perturbations are sufficiently small, they do not significantly change the system's dynamics, but enable the intrinsic system dynamics to drive the trajectory to the desired state. The Macau an Grebogi method is subdivided into two sequential parts: in the first part, the points p_{xs} and p_{xt} are found, so that there is a real orbit that goes from p_{xs} and p_{xt}, and in the second part, this orbit is used to build a constructive virtual orbit that allows the transfer from p_{xs} and p_{xt} using smaller number of elements of real orbits. In this process, small perturbations to the control parameter are used to move among the real orbits.

The effect of these perturbations is to change the system's evolution from one real orbit to another, resulting in a constructive orbit that allows the transfer from p_{xs} and p_{xt} at a faster time than the first part of our method. Thus, the overall effect of this procedure is to produce a sub-optimal solution that is obtained by the elimination of parts of the orbit, where recurrences occur with the use of small perturbations.

3. **Results**

The goal of smart structures technology is to reproduce biological functions in structural systems subjected to certain external loadings. These biological functions include a shape or skeletal system to provide the strength and fatique properties, a nervous system represented by sensors to monitor the state of the structure, a motor system to provide the optimal response, an immune system to provide capability, and a neural system to provide learning and

decision (Jain and Sirkis 1994; Mihailescu and Chiroiu 2004). Smart materials respond to external solicitations by changing their properties (for example the multifunctional carbon nanotube foils (Beldiman, Munteanu and Poienariu 2010)). Depending on the boundary given data, the mechanical waves migrate into the material and as response; the material is finding by itself the minimum length path for wave motion. Often, the reconstructions from measurement data are easier for smart materials (Munteanu, Delsanto and Poienariu, 2011).

Let us consider a simple piece of prismatic membrane (Fig.2). The Turing procedure is able to fill the interior of this membrane with a structured material able of taking over the mechanical actions mainly by axial forces. We refer to the shape memory alloys that mimic the behaviour of biological organisms such as fish or insects (Penrod et al. 2002).

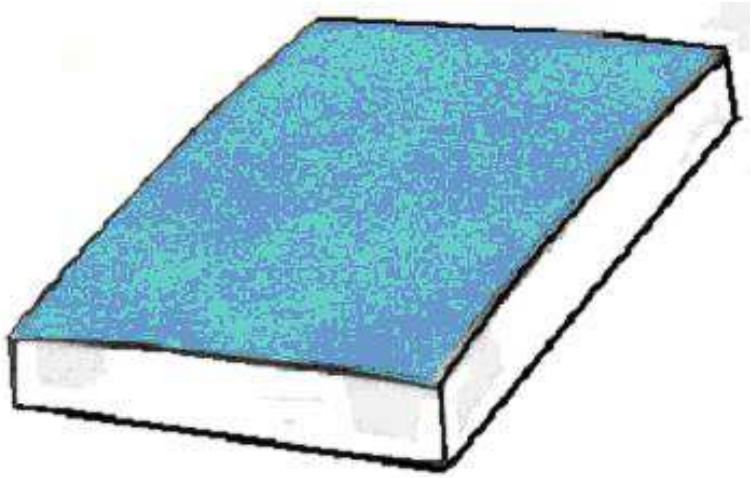

Fig.2. Prismatic domain.

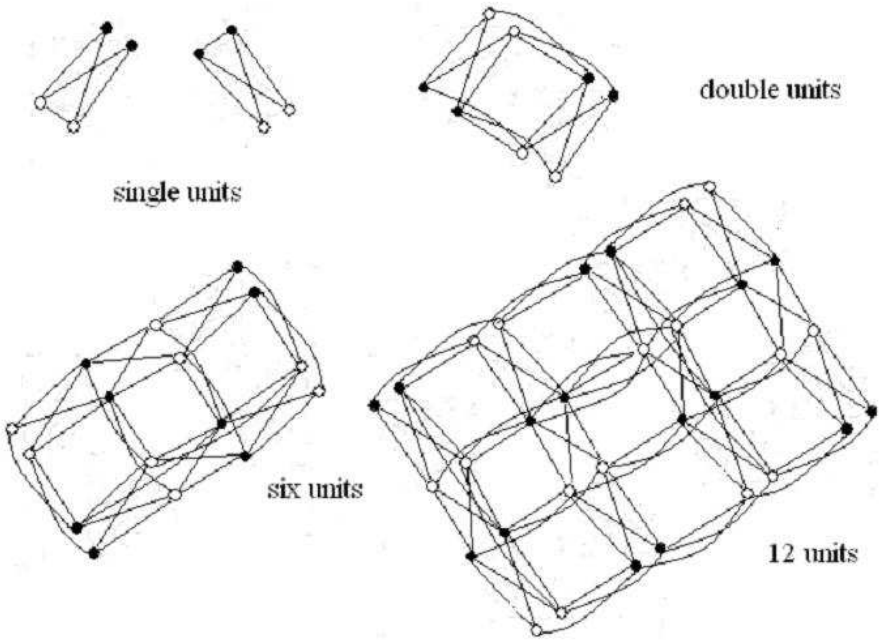

Fig.2.1 Growth of an internal complex structure from a single unit.

The ability of shape memory alloys to change shape in response to thermal or electrical stimuli considerably simplifies construction of biomimetic systems. The functions $u(x,t)$ and $v(x,t)$ will measure here the austenite and martensite phases fractions, respectively. The growth of this internal complex structure from a single unit, viewed as a perturbation, is displayed in Fig.3, for $\alpha_1 = 0.5$, $\alpha_2 = 1$, $\beta_1 = 0.1$, $\beta_2 = 2.4$, $-\alpha_3 = \beta_3 = 0.02$, $-\alpha_4 = \beta_4 = 3.1$, $-\alpha_5 = \beta_5 = 0.2$. The perturbation is done by an axial excitation.

The new material is able to form various shells concepted as solid continua made of small prismatic elements with two infinitesimal sides, inscribed on its middle surface and the third one, equal to the shell thickness, as shown in Fig.3 (Mihailescu and Chiroiu 2004).

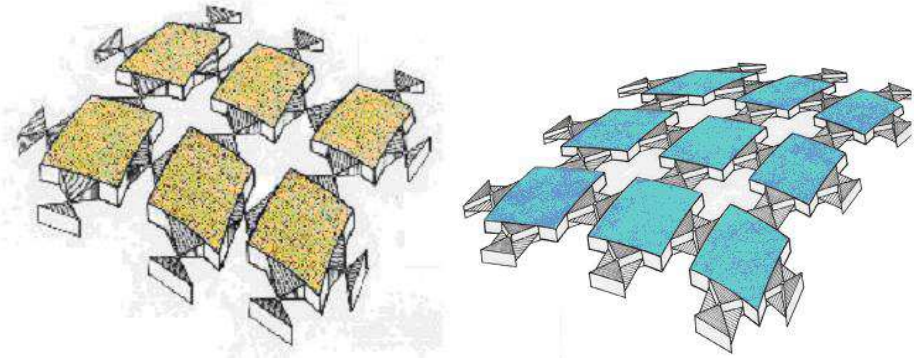

Fig. 3. Complex shell structures (Mihailescu and Chiroiu 2004).

In stability of the process is analyzed next, and simulated multi-periodical trajectories in the phase space, are displayed in fig. 4, for 1000 forcing period. The unstable periodic solutions u and v become stable after $t = 100s$.

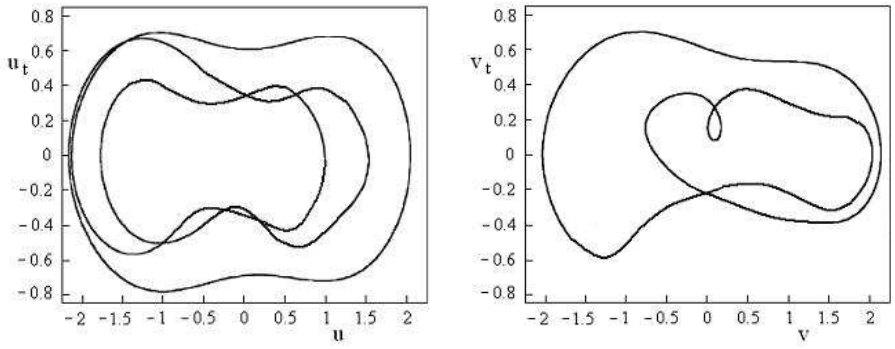

Fig. 4. Phase portraits.

4. Conclusions

Smart materials are characterized by an increasing of their mechanical properties and intelligence in a predictable way, due to the phase transition, as response to the external loading. This paper apply the self-organizing Turing pattern to create inhomogeneities that are leading to stationary spatial structure. For the structures that do not have an equilibrium state, the particular patterns that are formed depend on the initial conditions. This means one can obtain a great deal of variety in one simulation. Furthermore, the patterns also depend on the geometry of the surface.

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