AN ALGORITHM FOR THE OPTIMAL SYNTHESIS OF A GUIDING MECHANISM

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Abstract: The aim of this paper is to give a different method from the classic ones for the synthesis of the four-bar guiding mechanism. The method consists of the decomposition of the mechanism into small parts, simple kinematic chains, which are synthesized separately and then we put together the results, obtaining the synthesis of the whole mechanism.

Key words: optimal synthesis**,** guiding mechanism synthesis, simple cinematic chain.

AMS classification: 82-04, 93B50, 65K10

1. **Introduction**

In this paper we present the synthesis of the four-bar guiding mechanism, by its decomposition into simple kinematic chains.

This kind of synthesis was suggested for the first time by I.I. Artobolevschi [1], who maintained that the synthesis of the "parts" of one mechanism is simpler and quicker than the classic method.

This method is very appropriate for complex mechanisms (with many elements and many kinematic couplings).

The four-bar guiding mechanism can be decomposed into two simple open kinematic chains: kinematic chain I, $A_0A_{1i}M_i$ and kinematic chain II, $B_0B_{1i}A_{1i}M_i$, **Figure 1**.

1.1. *Kinematic chain I*, $A_0A_{1I}M_I$

It is an open kinematic chain and contains two elements: $A_0A_{1i} = a$ and $A_{1i}M_i = e$. Giving the coordinates X_i, Y_i of the points M_i on the trajectory "mm" and the corresponding angles α_I, we have to determine the optimum values of the lengths a and e and the coordinates X_{A0}, Y_{A0} of the fixed joint A_0.

1.2. *Kinematic chain II*, $B_0B_{1I}A_{1I}M_I$

It is an open kinematic chain with two elements. We know the coordinates X_i and Y_i and the angles ψ_i^* for N random points M_i on the curve "mm". We have to determine 5 unknown elements: the lengths *c* and *b*, the angle γ and the coordinates X_{B0}, Y_{B0} of the joint B_0.

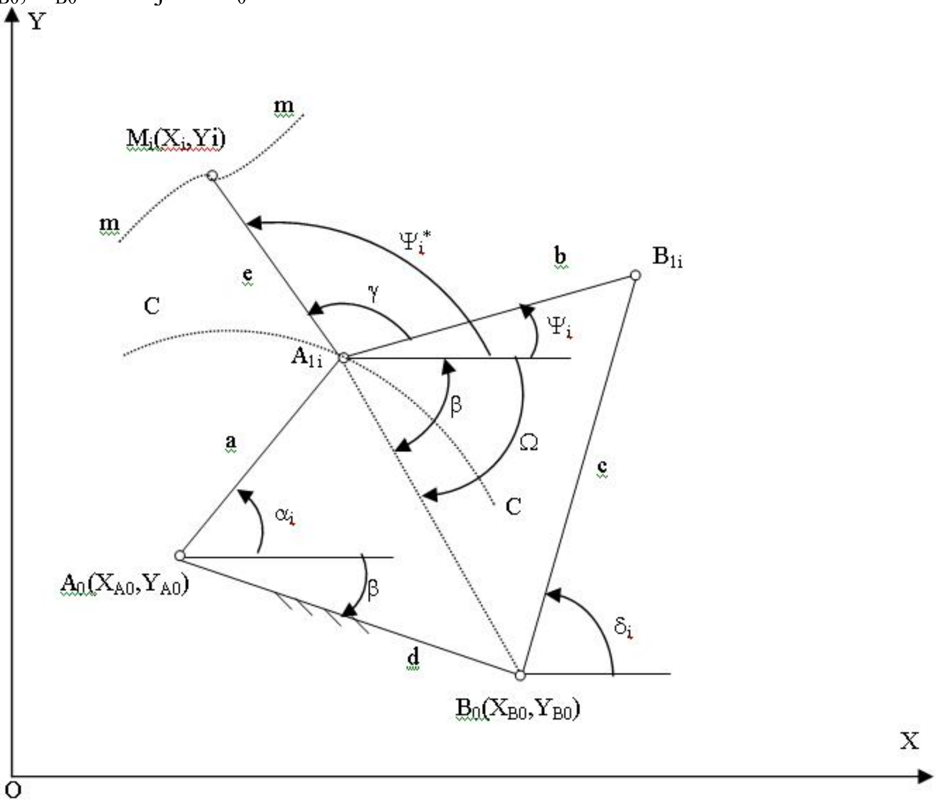

Figure 1. *The guiding mechanism*

In conclusion, for this guiding mechanism which generates trajectories, we have 9 unknown elements (4 for the kinematic chain $A_0A_{1i}M_i$ and 5 for the kinematic chain $B_0B_{1i}A_{1i}M_i$).

2. **Mechanism synthesis**

2.1. *The synthesis of kinematic chain I* ($A_0A_{1I}M_I$)

The coordinates of the point M (see **Figure 2**) are given by expressions:

$$X_{Mi}=X_{A0} + a \cos \alpha_i + e \cos \psi_i^* \quad (1)$$
$$Y_{Mi}=Y_{A0} + a \sin \alpha_i + e \sin \psi_i^* \quad (2)$$

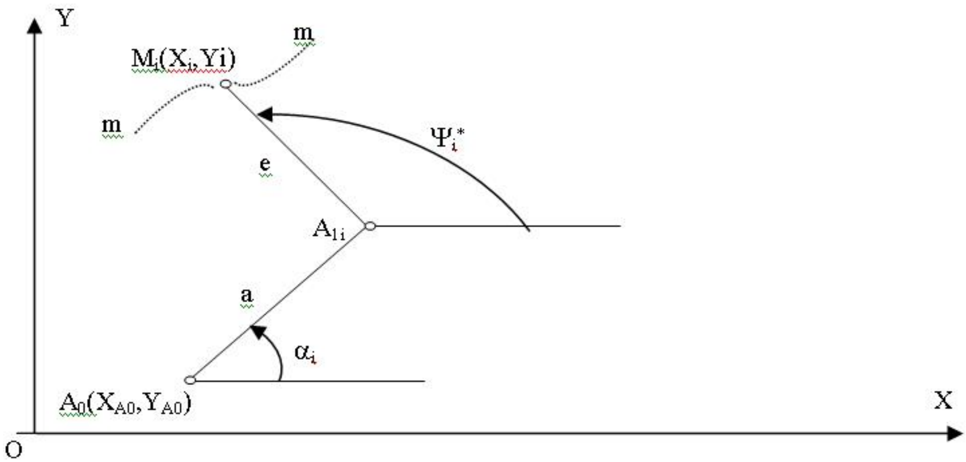

Figure 2. *Kinematic chain I*

where the elements X_{Mi}, Y_{Mi} and α_i are known, and the unknown elements are a, e, X_{A0}, Y_{A0}. In the relations (1) and (2) we remove the angle ψ_i^* and we obtain:

$$e^2 = Y_{Mi}^2 + Y_{A0}^2 + a^2 \sin^2\alpha_i - 2Y_{Mi}Y_{A0} + 2Y_{A0}\, a\sin \alpha_i - 2Y_{Mi}\, a \sin\alpha_i + X_{Mi}^2 + X_{A0}^2 + a^2\cos^2\alpha_i - 2X_{Mi}X_{A0} + 2X_{A0}\, a \cos \alpha_i - 2X_{Mi}\, a \cos \alpha_i$$

or

$$-(X_{Mi}^2 + Y_{Mi}^2) = X_{A0}^2 + Y_{A0}^2 + a^2 - e^2 - 2 X_{Mi}X_{A0} - 2Y_{Mi}Y_{A0} - 2a(Y_{Mi} \sin\alpha_i + X_{Mi} \cos\alpha_i) + 2aY_{A0} \sin\alpha_i + 2aX_{A0} \cos\alpha_i \quad (3)$$

We make the following notations:

$$\begin{cases} X_1 = X_{A0}^2 + Y_{A0}^2 + a^2 - e^2 \\ X_2 = X_{A0} \\ X_3 = Y_{A0} \\ X_4 = a \\ X_5 = aY_{A0} = X_3X_4 \\ X_6 = aX_{A0} = X_2X_4 \end{cases} \quad (4)$$

and

$$\begin{cases} A_{1i} = 1 \\ A_{2i} = -2X_{Mi} \\ A_{3i} = -2\, Y_{Mi} \\ A_{4i} = -2\, (Y_{Mi} \sin \alpha_i + X_{Mi} \cos \alpha_i) \\ A_{5i} = 2 \sin \alpha_i \\ A_{6i} = 2 \cos \alpha_i \\ B_i = -(X_{Mi}^2 + Y_{Mi}^2) \end{cases} \quad (5)$$

The relation (3), using the notations (4) and (5), becomes:

$$A_{1i}X_1 + A_{2i}X_2 + A_{3i}X_3 + A_{4i}X_4 + A_{5i}X_5 + A_{6i}X_6 = B_i \quad (6)$$

The synthesized kinematic chain doesn't generate exactly the trajectory function in every position i = 1…n, but approximates it. Thus the expression (6) can be rewritten in the following way:

$$P_i = \sum_{j=1}^{6} A_{ji} X_j - B_i, \; i = 1,..., N \tag{7}$$

The expression (7) represents the approximation error, which has to have the smallest value. The expression (7) is called the synthesis equation of the kinematic chain I.

We have to find the parameters X_i of the kinematic chain, for which the error P_i is maintained at an acceptable level when the independent variable α_i takes values in its variation interval.

The optimum dimensions of the kinematic chain which comply with the condition that the error should have the lowest value are those for which the expressions

$$I = \sum_{i}^{N} P_i^2 = \sum_{i=1}^{N} \left(\sum_{j=1}^{6} A_{ji} X_j - B_i\right)^2 \rightarrow \min \tag{8}$$

$$X_5 - X_3X_4 = 0 \tag{9}$$

$$X_6 - X_2X_4 = 0 \tag{10}$$

are simultaneously fulfilled.

We define the Lagrange function:

$$L = I + \lambda_1(X_5 - X_3X_4) + \lambda_2(X_6 - X_2X_4) \tag{11}$$

and we set the condition that the partial derivatives $\partial L/\partial X_i$, i=1,6, have to be zero:

$$\begin{cases} \dfrac{\partial L}{\partial X_1} = 2\sum_{i=1}^{N} (-B_i + \sum_{j=1}^{6} A_{ji} X_j) A_{1i} = 0 \\ \dfrac{\partial L}{\partial X_2} = 2\sum_{i=1}^{N} (-B_i + \sum_{j=1}^{6} A_{ji} X_j) A_{2i} - \lambda_2 X_4 = 0 \\ \dfrac{\partial L}{\partial X_3} = 2\sum_{i=1}^{N} (-B_i + \sum_{j=1}^{6} A_{ji} X_j) A_{3i} - \lambda_1 X_4 = 0 \\ \dfrac{\partial L}{\partial X_4} = 2\sum_{i=1}^{N} (-B_i + \sum_{j=1}^{6} A_{ji} X_j) A_{2i} - \lambda_1 X_3 - \lambda_2 X_2 = 0 \\ \dfrac{\partial L}{\partial X_5} = 2\sum_{i=1}^{N} (-B_i + \sum_{j=1}^{6} A_{ji} X_j) A_{5i} + \lambda_1 = 0 \\ \dfrac{\partial L}{\partial X_6} = 2\sum_{i=1}^{N} (-B_i + \sum_{j=1}^{6} A_{ji} X_j) A_{6i} + \lambda_2 = 0 \end{cases} \tag{12}$$

Obs. λ_1 and λ_2 are Lagrange multipliers.

The system (6) can be written in a matricial way:
-the coefficient matrix:

$$[A]=\begin{bmatrix} \sum_{i=1}^{N} A_{1i}^2 & \sum_{i=1}^{N} A_{2i}A_{1i} & \sum_{i=1}^{N} A_{3i}A_{1i} & \cdots & \cdots & \sum_{i=1}^{N} A_{6i}A_{1i} \\ \sum_{i=1}^{N} A_{1i}A_{2i} & \sum_{i=1}^{N} A_{2i}^2 & \sum_{i=}^{N} A_{3i}A_{2i} & \cdots & \cdots & \sum_{i=1}^{N} A_{6i}A_{2i} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sum_{i=1}^{N} A_{1i}A_{6i} & \sum_{i=1}^{N} A_{2i}A_{6i} & \sum_{i=1}^{N} A_{3i}A_{6i} & \cdots & \cdots & \sum_{i=1}^{N} A_{6i}^2 \end{bmatrix} \tag{13}$$

Obs. The elements of matrix A are obtained according to the rules:

$$a_{jk} = \sum_{i=1}^{N} A_{ji}A_{ki} \qquad ;\ j \neq k;\ j,\ k = 1,2,\ldots,6$$

$$a_{jk} = \sum_{i=1}^{N} A_{ji}^2 \qquad ;\ j = k;\ j,\ k = 1,2,\ldots,6$$

-the variables matrix,

$$[X]^T = [X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5 \quad X_6]^T \tag{14}$$

-the constant terms

$$[B]=\begin{bmatrix} \sum_{i=1}^{N} B_i A_{1i} & 0 & 0 \\ \sum_{i=1}^{N} B_i A_{2i} & 0 & \frac{X_4}{2} \\ \sum_{i=1}^{N} B_i A_{3i} & \frac{X_4}{2} & 0 \\ \sum_{i=1}^{N} B_i A_{4i} & \frac{X_3}{2} & \frac{X_2}{2} \\ \sum_{i=1}^{N} B_i A_{5i} & -\frac{1}{2} & 0 \\ \sum_{i=1}^{N} B_i A_{6i} & 0 & -\frac{1}{2} \end{bmatrix} \tag{15}$$

Using notations (13), (14) and (15), the system (12) can be rewritten in this way:

$$[A]\cdot[X] = [B]\cdot\begin{bmatrix} 1 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} \tag{16}$$

By the simultaneously solving of the equations (10) and (16), we obtain the unknown elements X_j; j=1, 6, i.e. the dimensional parameters of the kinematic chain.

2.2 *The synthesis of kinematic chain II* ($B_0B_{1I}A_{1I}M_I$)

The unknown geometrical parameters are: *b*, *c*, X_{B0}, Y_{B0} and (the angle) γ.

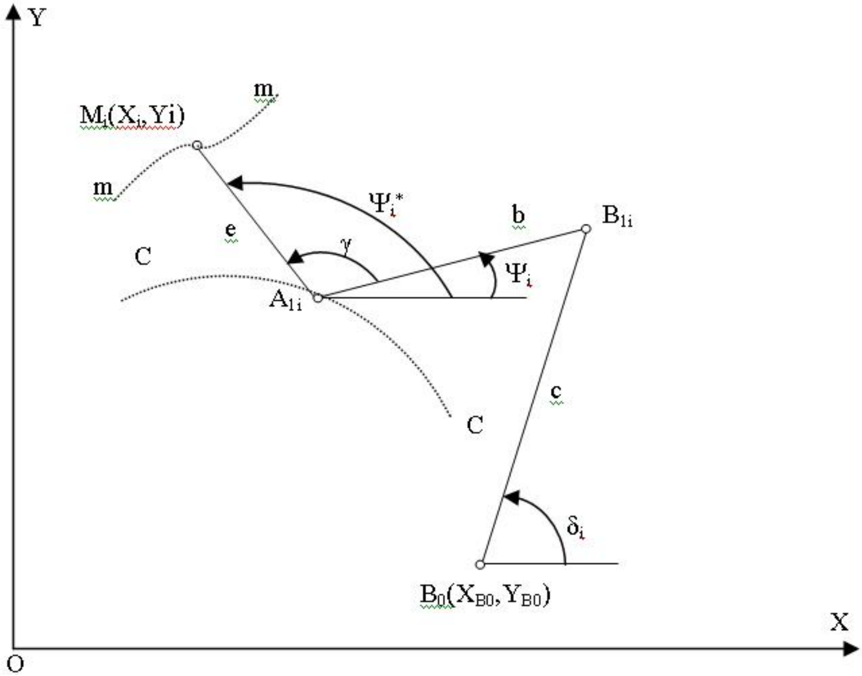

Figure 3. *Kinematic chain II*

We consider as known parameters the next parameters: X_{A1i}, Y_{A1i} and ψ_i^* (from solving the synthesis of the kinematic chain $A_0A_{1i}M_i$).

The synthesis equation for this kinematic chain can be obtained writing the relations (see **Figure 3**):

$$X_{B1i}=X_{A1i} + b \cos (\psi_I^* - \gamma) \tag{17}$$

$$Y_{B1i}=Y_{A1i} + b \sin (\psi_i^* - \gamma) \tag{18}$$

$$(X_{B1i} - X_{B0})^2 + (Y_{B1i} - Y_{B0})^2 = c^2 \tag{19}$$

By expanding, we obtain the relation:

$2b\sin\gamma(X_{A1i}\sin\psi_i^* - Y_{A1i}\cos\psi_i^*) + 2b\cos\gamma(X_{A1i}\cos\psi_i^* + Y_{A1i}\sin\psi_i^*) - 2X_{A1i}X_{B0} - 2Y_{A1i}Y_{B0} + X_{B0}^2 + Y_{B0}^2 + b^2 - c^2 - 2\cos\psi_i^*(bX_{B0}\cos\gamma - bY_{B0}\sin\gamma) - 2\sin\psi_i^*(bY_{B0}\cos\gamma + bX_{B0}\sin\gamma) = -(X_{A1i}^2 + Y_{A1i}^2)$ (20)

We make the notations:

$$\begin{cases} X_1 = b\sin\gamma \\ X_2 = b\cos\gamma \\ X_3 = X_{B0} \\ X_4 = Y_{B0} \\ X_5 = X_{B0}^2 + Y_{B0}^2 + b^2 - c^2 \\ X_6 = X_{B0}b\cos\gamma - Y_{B0}b\sin\gamma = -(X_1X_4 - X_2X_3) \\ X_7 = Y_{B0}b\cos\gamma - X_{B0}b\sin\gamma = X_1X_3 + X_2X_4 \end{cases} \quad (21)$$

$$\begin{cases} A_{1i} = 2(X_{A1i}\sin\psi_I^* - Y_{A1i}\cos\psi_i^*) \\ A_{2i} = 2(X_{A1i}\cos\psi_I^* + Y_{A1i}\sin\psi_i^*) \\ A_{3i} = -2X_{A1i} \\ A_{4i} = -2Y_{A1i} \\ A_{5i} = 1 \\ A_{6i} = 2\cos\psi_i^* \\ A_{7i} = -2\sin\psi_i^* \end{cases} \quad (22)$$

$$B_i = -(X_{A1i}^2 + Y_{A1i}^2) \quad (23)$$

By using relations (21), (22) and (23) in the relation (20), we obtain:

$$A_{1i}X_1 + A_{2i}X_2 + A_{3i}X_3 + A_{4i}X_4 + A_{5i}X_5 + A_{6i}X_6 + A_{7i}X_7 = B_i \text{ or}$$

$$\sum_{j=1}^{7}(A_{ji}X_j - B_i) = 0 \qquad , i = 1, N \quad (24)$$

The relation (24), is called the synthesis equation of the kinematic chain II ($B_0B_{1i}A_{1I}M_i$), in which the unknown elements X_j; j = 1,7 contain the geometrical parameters (5 parameters) which define the chain and the coefficients A_{ji} şi B_i are functions.

In the equation (24), the unknown elements X_1, X_2, X_3, X_4, X_5 are independent and the unknown elements X_6 and X_7 are functions of the first five elements.

The geometrical parameters which define the optimum kinematic chain that id to generate the prescribed curve “mm” can be obtained by simultaneously solving the following equations:

$$I = \sum_{i}^{N} P_i^2 = \sum_{i=1}^{N} (\sum_{j=1}^{7} A_{ji}X_j - B_i)^2 \rightarrow \min \quad (25)$$

$$X_6 + X_1X_4 - X_2X_3 = 0 \quad (26)$$

$$X_7 - X_1X_3 - X_2X_4 = 0 \quad (27)$$

For this reason, we define the Lagrange function:

$$L1 = I + \lambda_1(X_6 + X_1X_4 - X_2X_3) + \lambda_2(X_7 - X_1X_3 - X_2X_4) \tag{28}$$

for which we set the condition that its partial derivatives have to be zero:

$$\partial L1/\partial X_i=0,\ i=1,7 \tag{29}$$

We obtain the following system:

$$[A]\cdot[X]=[B]\cdot\begin{bmatrix}1\\ \lambda_1\\ \lambda_2\end{bmatrix} \tag{30}$$

where:

- [A] is the coefficient symmetric 7x7 matrix :

$$[A]=[a_{jk}]=\left[\sum_{i=1}^{N}A_{ji}A_{ki}\right] \qquad ;j,k=1,2,...,7 \tag{31}$$

- [X] is the unknown elements matrix:

$$[X]^T=[X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5 \quad X_6 \quad X_7]^T \tag{32}$$

- [B] is the 7x3 known elements matrix:

$$[B]=\begin{bmatrix}
\sum_{i=1}^{7}A_{1i}B_i & -\frac{X_4}{2} & \frac{X_3}{2}\\
\sum_{i=1}^{7}A_{2i}B_i & \frac{X_3}{2} & \frac{X_4}{2}\\
\sum_{i=1}^{7}A_{3i}B_i & \frac{X_2}{2} & \frac{X_1}{2}\\
\sum_{i=1}^{7}A_{4i}B_i & -\frac{X_1}{2} & \frac{X_2}{2}\\
\sum_{i=1}^{7}A_{5i}B_i & 0 & 0\\
\sum_{i=1}^{7}A_{6i}B_i & -\frac{1}{2} & 0\\
\sum_{i=1}^{7}A_{7i}B_i & 0 & -\frac{1}{2}
\end{bmatrix} \tag{33}$$

The systems (12) and (30) can be solved by iterations, for different values of the Lagrange multipliers λ_1 and λ_2, starting with $\lambda_1 = \lambda_2 = 0$.

For $\lambda_1 = \lambda_2 = 0$, the systems (12) and (30) become linear systems, with solutions $X_j^{(0)}$; j=1,6 and j=1,7 respectively.

By substituting the solutions $X_j^{(0)}$ into the relations (8) and (25) respectively we obtain $I^{(0)}$, then we introduce the solutions into the matrix [B] (for each chain), which are solved again in order to obtain all the solutions:

$$X_j = \sum_{l=1}^{2} C_{jl}\lambda_l + C_{j0} \quad ; j = 1{,}6,\ j = 1{,}7 \ \text{ respectively} \tag{34}$$

Obs. The coefficients C_{jl}, j=1,6 and j=1,7 respectively; l=0,1,2 are known in the relation (34).

We use the solutions X_j obtained from the relations (34) into the equations (9), (10) and (26),(27) respectively. Thus we obtain two second degree equations in λ_1 and λ_2:

$$\begin{aligned} &\lambda_1^2 + g_{11}\lambda_1\lambda_2 + g_{12}\lambda_2^2 + g_{13}\lambda_1 + g_{14}\lambda_2 + g_{15} = 0 \\ &\lambda_1^2 + g_{21}\lambda_1\lambda_2 + g_2\lambda_2^2 + g_{23}\lambda_1 + g_{24}\lambda_2 + g_{25} = 0 \end{aligned} \tag{35}$$

From the equations (35) we remove λ_2 (or λ_1) and we obtain a fourth degree equation in λ_1 (or λ_2):

$$h_0\lambda_1^4 + h_1\lambda_1^3 + h_2\lambda_1^2 + h_3\lambda_1 + h_4 = 0 \tag{36}$$

In the relations (35) and (36) the coefficients g_{jk} (j=1,2; k=1,…,5) and h_i (i=1,…,4) are known.

The equations (35) are fulfilled by four pairs of real values λ_1 and λ_2. For each pair, from the equations (35) we obtain the solutions set $X_j^{(1)}$ of the first iteration. Then we put these values into the equations (9),(10) and (26),(27) respectively and we obtain the values $I^{(1)}$. If $I^{(1)}<\varepsilon$ or $(I^{(1)}- I^{(0)}) < \varepsilon$, we can consider that the optimum dimensions of the kinematic chain are determined (the relations (8) and (25) respectively). If $(I^{(1)}- I^{(0)}) > \varepsilon$ we make the second iteration, by putting the solutions $X_j^{(1)}$ (which minimize $I^{(1)}$) into the matrix B. This way we obtain the solutions for the second iteration.

One or three iterations are required in practical cases, so the method is very quick and can be implemented on computer.

3. **Summary**

The advantages of the method are:

- It eliminates the necessity of choosing initial values;
- The calculus can be made on computer;
- It has no restrictions regarding the number of points through which the curve « mm » is plotted.

The disadvantage of the method is the estimation of Lagrange multiplicators. These estimations interfere in defining the searching direction and determine the convergence velocity [4].

Not all synthesized mechanisms are technological mechanisms.

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