Neutrosophic Computational Models – I

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Abstract

Neutrosophic approaches in logics and computing have been proposed by Smarandache and interesting developments were obtained by many researchers around the world. Not only algebraic structures, topology, statistics, and logics already benefited by the new interpretation, but also many fields of computer science: image analysis, neutrosophic databases, neutrosophic cognitive maps etc. This paper describes some neutrosophic computational models in order to identify a set of requirements for software implementation.

1. Introduction

There are various fields where the models developed by scientists contain parameters affected by incomplete coverage (knowledge missing), uncertainty (random factors) or imprecision (measurements, linguistic variables).

Neutrosophic logic is a mathematical model of uncertainty on non-Archimedean structures being able to measure simultaneously the truth, indeterminacy, and falsity as described in [20, 21, 25, 26], to mention only a few references.

The neutrosophic approach already proved its value both in theoretical and applied fields. Vasantha Kandasamy W.B. and Smarandache F. [16] have defined neutrosophic numbers and they developed specific theories on various neutrosophic algebraic structures. Finite neutrosophic complex numbers, neutrosophic topological spaces [19], and neutrosophic probability [21, 14] are some pure mathematical developments. Also computer science structures like neutrosophic automata [29], and neutrosophic graphs [15] were proposed to be used for suitable applications.

Neutrosophic models have been developed for options market [7], financial market [17], image denoising [13, 18], information retrieval [1, 2, 3, and

11], intelligent information processing by neutrosophic cognitive maps [9, 12], robotics [32], physics [28] etc.

Many applications are extensions of fuzzy models or intuitionistic fuzzy (Atanassov) models [8]. The present material continues with the following structure: the neutrosophic approaches (neutrosophic sets, neutrosophic logics, various neutrosophic operations) are presented in the next section, while the neutrosophic relations and appropriate data structures are developed in the third section.

2. Neutrosophic approaches

Neutrosophic sets/logics have been introduced by Smarandache [21, 22] as a generalization of the *Intuitionistic Fuzzy Sets/Logics* models proposed by Atanassov [5].

Neutrosophic approaches consider the sets T, I, and F as being standard or non-standard real subsets of $]^-0, 1^+[$, where $1^+ = 1 + \epsilon$, $^-0 = 1 - \epsilon$, with $\epsilon > 0$ an infinitesimal, that is $|\epsilon| < 1/n$ for all positive integers n, and sup and inf are known for every one of these sets: $\sup T = t_{\sup}$, $\inf T = t_{\inf}$, $\sup I = i_{\sup}$, $\inf I = i_{\inf}$, $\sup F = f_{\sup}$, $\inf F = f_{\inf}$, with

$$^-0 \leq t_{\sup} + i_{\sup} + f_{\sup} \leq 3^+.$$

The sets T, I, and F are the *neutrosophic components* and represent the membership/truth value, indeterminacy value, and non-membership/falsehood value for a given set A of universe U, or a proposition p.

If T, I, and F are subintervals of $]^-0, 1^+[$, the neutrosophic set A is an interval neutrosophic set (INS). For real scientific and engineering applications it is assumed that T, I, and F are intervals of $[0, 1]$. As described by Wang et al. [33] and Schumann [20], Smarandache's model generalizes all previously defined models (see Table 1)): classical sets, FS – Fuzzy Sets [34], IVFS – Interval Valued Fuzzy Sets (see references mentioned by Dubois et al. [10]), AIFS – Atanassov Intuitionistic Fuzzy Sets [5], AIVIFS – Atanassov Interval Valued Intuitionistic Fuzzy Sets[6], PS – Paraconsistent Sets [21], and IVPS – Interval Valued Paraconsistent Sets [21, 20].

Table 1: Various models of sets

Model	T_A	I_A	F_A	Constraints
Classic	$T_A(u) = \alpha \in \{0,1\}$	$I_A(u) = 0$	$F_A(u) = \beta \in \{0,1\}$	$\alpha + \beta = 1$
FS	$T_A(u) = \alpha \in [0,1]$	$I_A(u) = 0$	$F_A(u) = \beta \in [0,1]$	$\alpha + \beta = 1$
IVFS	$T_A(u) = [t_{\min}, t_{\max}]$ $t_{\min}, t_{\max} \in [0,1]$	$I_A(u) = 0$	$F_A(u) = [f_{\min}, f_{\max}]$ $f_{\min}, f_{\max} \in [0,1]$	$t_{\max} + f_{\min} = 1,$ $t_{\min} + f_{\max} = 1$
AIFS	$T_A(u) = \alpha \in [0,1]$	$I_A(u) = 0$	$F_A(u) = \beta \in [0,1]$	$\alpha + \beta \leq 1$
AIVIFS	$T_A(u) = [t_{\min}, t_{\max}]$ $[t_{\min}, t_{\max}] \subset [0,1]$	$I_A(u) = 0$	$F_A(u) = [f_{\min}, f_{\max}]$ $[f_{\min}, f_{\max}] \subset [0,1]$	$t_{\max} + f_{\min} \leq 1$
PS	$T_A(u) = \alpha \in [0,1]$	$I_A(u) = 0$	$F_A(u) = \beta \in [0,1]$	$\alpha + \beta > 1$
IVPS	$T_A(u) = [t_{\min}, t_{\max}]$ $[t_{\min}, t_{\max}] \subset [0,1]$	$I_A(u) = 0$	$F_A(u) = [f_{\min}, f_{\max}]$ $[f_{\min}, f_{\max}] \subset [0,1]$	$t_{\max} + f_{\min} > 1$

The classical set operations like union, intersection, difference, complement and cartesian product are well known. Let X and Y be two real stan-

dard or non-standard subsets included in the non-standard interval $]^-0, \infty)$. In table 2 there are described the following operations: addition, subtraction, multiplication, and division by a number [23, 33, 27].

Table 2: Computing with sets

Operation	Symbol	Definition	Observation
Addition	$\oplus$	$X \oplus Y = \{s \mid s = x + y, x \in X, y \in Y\}$	$\inf\{X \oplus Y\} = \inf X + \inf Y,$ $\sup\{X \oplus Y\} = \sup X + \sup Y$
Subtraction	$\ominus$	$X \ominus Y = \{s \mid s = x - y, x \in X, y \in Y\}$	$\inf\{X \ominus Y\} = \inf X - \sup Y,$ $\sup\{X \ominus Y\} = \sup X - \inf Y$
Multiplication	$\otimes$	$X \otimes Y = \{s \mid s = x \times y, x \in X, y \in Y\}$	$\inf\{X \otimes Y\} = \inf X \times \inf Y,$ $\sup\{X \otimes Y\} = \sup X \times \sup Y$
Division	$\oslash$	$Y = \{y\} \subset (1, \infty)$ $X \oslash y = \{s \mid s = x/y, x \in X\}$	$\inf\{X \oslash y\} = \inf X/y,$ $\sup\{X \oslash y\} = \sup X/y$

Let X and Y be neutrosophic sets of the universe U. Let (T_X, I_X, F_X), respective (T_Y, I_Y, F_Y) be the corresponding neutrosophic components. Based on operations described in Table 2, the basic neutrosophic set operations can be defined as in Table 3. If a negative result or greater than 1 value will be obtained after calculations (according to the computing formula) the result should be replaced with $^-0$ or 1^+, respectively.

Table 3: Basic Neutrosophic Set Operations [21, 23, 26, 27, 14]

Operation	Symbol	Definition
Union	$\cup$	$T_{X \cup Y} = (T_X \oplus T_Y) \ominus (T_X \otimes T_Y)$ $I_{X \cup Y} = (I_X \oplus I_Y) \ominus (I_X \otimes I_Y)$ $F_{X \cup Y} = (F_X \oplus F_Y) \ominus (F_X \otimes F_Y)$
Intersection	$\cap$	$T_{X \cap Y} = T_X \otimes T_Y$ $I_{X \cap Y} = I_X \otimes I_Y$ $F_{X \cap Y} = F_X \otimes F_Y$
Difference	$\backslash$	$T_{X \backslash Y} = T_X \ominus (T_X \otimes T_Y)$ $I_{X \backslash Y} = I_X \ominus (I_X \otimes I_Y)$ $F_{X \backslash Y} = F_X \ominus (F_X \otimes F_Y)$
Complement of X	$C(X)$	$T_{C(X)} = F_X$ or $T_{C(X)} = \{1^+\} \ominus T_X$ [23, 27] $I_{C(X)} = I_X$ [14] or $I_{C(X)} = \{1^+\} \ominus I_X$ [23, 27] $F_{C(X)} = T_X$ or $F_{C(X)} = \{1^+\} \ominus F_X$ [23, 27].

A partial order relation for the neutrosophic set/logic approaches can be defined according to [4, 26]: $X \subset Y$ if and only if $T_X(u) \leq T_Y(u)$, $I_X(u) \geq I_Y(u)$, and $F_X(u) \geq F_Y(u)$, $\forall u \in U$, for crisp components. For general neutrosophic components the definition can be extended as follows:

$$X \subset Y \quad \text{if and only if} \quad \begin{aligned} &\inf T_X \leq \inf T_Y, \sup T_X \leq \sup T_Y, \\ &\inf I_X \geq \inf I_Y, \sup I_X \geq \sup I_Y, \text{and} \\ &\inf F_X \geq \inf F_Y, \sup F_X \geq \sup F_Y. \end{aligned}$$

The last definition works for both crisp and general components.

The developments of fuzzy set/logic theory were based on triangular norms (for *intersection, conjunction, and* operations) and conorms (for *union, disjunction, or* operations).

Let a, b, c be any numbers in $[0, 1]$ as in fuzzy set theory. For neutrosophic set/logic approaches the N-norm (denoted by N_n), respective the N-conorm (denoted by N_c), have to satisfy the following axioms (Table 4). Table 5 gives the most known N-norms and N-conorms, as in fuzzy logic.

Table 4: Axioms to be satisfied by N-norms and N-conorms [26]

Property	N_n	N_c
Boundary conditions	$N_n(a, 0) = 0, N_n(a, 1) = a$	$N_c(a, 1) = 1, N_c(a, 0) = a$
Monotonicity	If $a \leq b$ then $N_n(a, c) \leq N_n(b, c)$	If $a \leq b$ then $N_c(a, c) \leq N_c(b, c)$
Commutativity	$N_n(a, b) = N_n(b, a)$	$N_n(a, b) = N_n(b, a)$
Associativity	$N_n(N_n(a, b), c) = N_n(a, N_n(b, c))$	$N_c(N_c(a, b), c) = N_c(a, N_c(b, c))$

Table 5: Most known N/t-norms and N/t-conorms [26, 31]

Operator	$N_n(a, b)/T(a, b)$	$N_c(a, b)/ \perp (a, b)$	Corresponding to
Min	$\min\{a, b\}$	$\max\{a, b\}$	$T_0, \perp_0$
Prod	ab	$a + b - ab$	$T_1, \perp_1$
Lukasiewicz	$\max\{0, a + b - 1\}$	$\min\{1, a + b\}$	$T_\infty, \perp_\infty$

Based on above settings and forwarding towards computational intelligent developments, let define more models of the neutrosophic components for: neutrosophic empty set, neutrosophic total (full) set, neutrosophic subset relation, neutrosophic union, neutrosophic intersection, neutrosophic difference, neutrosophic complement etc. For the universal set U let us denote $\emptyset_N$ any of the following neutrosophic models: $(\emptyset, \emptyset, 1)$, $(\emptyset, 1, 1)$, $(\emptyset, 1, \emptyset)$, and $(\emptyset, \emptyset, \emptyset)$, the first one being the most natural according to the usual way of thinking. Similarly, the total/full set U_N can be modelled by: $(1, \emptyset, \emptyset)$, $(1, 1, \emptyset)$, $(1, \emptyset, 1)$, and $(1, 1, 1)$. Any non-empty subset of U is modelled by (T, I, F), where T, I, and F are its neutrosophic components. However, a *neutrosophic classical set* A is defined by (T_A, I_A, F_A) with $T_A \cap I_A \cap F_A = \emptyset$, and T_A, I_A, and F_A as subsets of U [14].

Let X, and Y be two neutrosophic sets of U. The result of operation $X \cap Y$ can be defined according to:

$$(N_n^1(T_X(u), T_Y(u)), N_n^2(I_X(u), I_Y(u)), N_n^3(F_X(u), F_Y(u))),$$

where N_n^i, $i \in 1, 2, 3$ can be any N-norm given above. The result of operation $X \cup Y$ can be defined according to:

$$(N_c^1(T_X(u), T_Y(u)), N_c^2(I_X(u), I_Y(u)), N_c^3(F_X(u), F_Y(u))),$$

where N_c^i, $i \in 1, 2, 3$ can be any N-conorm already described. However, other models have been proposed and used both in theory and applications as shown in [26, 30, 33], and other works. An interesting proposal, by Smarandache [26], defines the principal neutrosophic set operations as in Table 6.

As shown in [33], if the neutrosophic components of the neutrosophic sets X and Y are intervals of $[0, 1]$, the neutrosophic set/logic operations can be defined according to Table 7.

Let N be a neutrosophic model given by $N = (T, I, F)$, where $T, I, F \subseteq [0, 1]$, and the set of syntactically well-formed formulas (in neutrosophic propositional/predicate calculus), as defined in [33]. A *neutrosophic valuation* associates to each formula p the triple representation $v(p) = (T, I, F)$ representing its truth degree, indeterminacy degree and falsity degree. However, the set I may be composed by many components like: indeterminacy, vagueness, imprecision, error, uncertainty, etc. Ashbacher [4] and Rivieccio [30] have considered the case of real values $(t, i, f) \in [0, 1]^3$ and analysed the neutrosophic connectives: conjunction, disjunction, negation, and implication. The case of interval neutrosophic logic was considered, in extension, in [33], while other models on non-Archimedean structures were considered by Schumann [20]. The most general case based on N-norms, and N-conorms can be discussed also based on developments provided by Smarandache in [26]. The synthesis of these results is shown in Table 8, where (N2), (C2), (D3), and (I2) were marked as best choice for neutrosophic valued logic, while (N2), (C2), (D3) and (I1) are used by intuitionistic neutrosophic logic [4], and (N3), (C3), (D3), and (I2) are used by interval neutrosophic logic [30, 33]. The paraconsistent neutrosophic logic is best represented by (N2), (C3), (D2), and (I2), according to [30].

Table 6: Smarandache' Neutrosophic Set Operations [26]

Operation	Definition: $\forall u \in U$
Intersection	$T_{X \cap Y}(u) = T_X(u)T_Y(u)$ $I_{X \cap Y}(u) = I_X(u)I_Y(u) + I_X(u)T_Y(u) + T_X(u)I_Y(u)$ $F_{X \cap Y}(u) = F_X(u)(T_Y(u) + I_Y(u) + F_Y(u)) + F_Y(u)T_X(u) + F_Y(u)I_X(u)$
Optimistic Intersection	$T_{X \cap Y}(u) = T_X(u)T_Y(u) + T_X(u)I_Y(u) + T_Y(u)I_X(u)$ $I_{X \cap Y}(u) = I_X(u)I_Y(u)$ $F_{X \cap Y}(u) = F_X(u)(T_Y(u) + I_Y(u) + F_Y(u)) + F_Y(u)T_X(u) + F_Y(u)I_X(u)$
Union	$T_{X \cup Y}(u) = T_X(u)(T_Y(u) + I_Y(u) + F_Y(u)) + T_Y(u)F_X(u) + T_Y(u)I_X(u)$ $I_{X \cup Y}(u) = I_X(u)I_Y(u) + I_X(u)F_Y(u) + I_Y(u)F_X(u)$ $F_{X \cup Y}(u) = F_X(u)F_Y(u)$
Pessimistic Union	$T_{X \cup Y}(u) = T_X(u)T_Y(u) + T_X(u)F_Y(u)) + T_Y(u)F_X(u)$ $I_{X \cup Y}(u) = I_X(u)(I_Y(u) + F_Y(u) + T_Y(u)) + I_Y(u)F_X(u) + I_Y(u)T_X(u)$ $F_{X \cup Y}(u) = F_X(u)F_Y(u)$

Table 7: Neutrosophic Set/Logic Operations [26, 33]

Operation	Definition: $\forall u \in U$
Intersection	$\inf T_{X \cap Y} = \min\{\inf T_X, \inf T_Y\}$, $\sup T_{X \cap Y} = \min\{\sup T_X, \sup T_Y\}$ $\inf I_{X \cap Y} = \max\{\inf I_X, \inf I_Y\}$, $\sup I_{X \cap Y} = \max\{\sup I_X, \sup I_Y\}$ $\inf F_{X \cap Y} = \max\{\inf F_X, \inf F_Y\}$, $\sup F_{X \cap Y} = \max\{\sup F_X, \sup F_Y\}$
Union	$\inf T_{X \cup Y} = \max\{\inf T_X, \inf T_Y\}$, $\sup T_{X \cup Y} = \max\{\sup T_X, \sup T_Y\}$ $\inf I_{X \cup Y} = \min\{\inf I_X, \inf I_Y\}$, $\sup I_{X \cup Y} = \min\{\sup I_X, \sup I_Y\}$ $\inf F_{X \cup Y} = \min\{\inf F_X, \inf F_Y\}$, $\sup F_{X \cup Y} = \min\{\sup F_X, \sup F_Y\}$
Complement	$T_{C(X)} = F_X$ $\inf I_{C(X)} = 1 - \sup I_X$, $\sup I_{C_X} = 1 - \inf I_X$ $F_{C(X)} = T_X$

If p belongs to interval neutrosophic predicate logic and uses the predicate P over a domain D under any of the quantifiers $\forall$, $\exists$, then

$$v(\forall x P) = (\inf T(P(x)), \inf I(P(x)), \sup F(P(x))), x \in D,$$

Table 8: Neutrosophic Logic Connectives: $v(p)$ is the neutrosophic valuation

Connectives	Definitions
Negation	(N1) v(¬p) = $(1-t, 1-i, 1-f)$
	(N2*) v(¬p) = (f,i,t)
	(N3) v(¬p) = $(f, 1-i, t)$
Conjunction	(C1) v(p ∧ q) = $(t_1 t_2, i_1 i_2, f_1 f_2)$
	(C2*) v(p ∧ q) = $(\min(t_1, t_2), \min(i_1, i_2), \max(f_1, f_2))$
	(C3) v(p ∧ q) = $(\min(t_1, t_2), \max(i_1, i_2), \max(f_1, f_2))$
Disjunction	(D1) v(p ∨ q) = $(t_1 + t_2 - t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2)$
	(D2) v(p ∨ q) = $(\max(t_1, t_2), \max(i_1, i_2), \min(f_1, f_2))$
	(D3*) v(p ∨ q) = $(\max(t_1, t_2), \min(i_1, i_2), \min(f_1, f_2))$
Implication	(I1) v(p→q) = $v(\neg p \wedge q)$
	(I2*) v(p→q) = $(\min(1, 1 - t_1 + t_2), \max(0, i_2 - i_1), \max(0, f_2 - f_1))$

and
$$v(\exists x P) = (\sup T(P(x)), \sup I(P(x)), \inf F(P(x))), x \in D,$$

with appropriate transformation to assure the consistency, according to the semantics of the neutrosophic predicate calculus [33].

3. **Computational approaches**

Vasantha Kandasamy W.B. and Smarandache F. [16] have been introduced the neutrosophic numbers and developed specific theories on various neutrosophic algebraic structures. For intelligent computational science structures like neutrosophic automata [29], and neutrosophic graphs [15] are very useful. Such entities will be discussed elsewhere.

In the following we consider neutrosophic relations as neutrosophic sets [33]. Let U and V be two crisp sets and $U \times V$ their cartesian product. A binary neutrosophic relation is a neutrosophic set S having the universe $U \times V$ and the neutrosophic components (T_S, I_S, F_S). As described above, the neutrosophic set operations can be applied to neutrosophic binary relations. Firstly, the case of valued neutrosophic binary relations will be presented. The composition of two neutrosophic binary relations R (on $U \times V$) and S (on $V \times W$) is the relation $R \circ S$ (on $U \times W$) having the neutrosophic components obtained as:

$$T_{R \circ S}(u, w) = \sup \{\min(T_R(u, v), T_S(v, w))| \text{ for all } v \in V\},$$

$$F_{R \circ S}(u, w) = \sup \{\min(F_R(u, v), F_S(v, w))| \text{ for all } v \in V\},$$

and
$$I_{R \circ S}(u, w) = \sup \{\min(I_R(u, v), I_S(v, w))| \text{ for all } v \in V\}.$$

If U, V, and W are finite crisp sets, then a matrix-based approach can be given. Every binary relation is defined by three matrices describing the neutrosophic components. Alternatively, a hyper-matrix can be used to define a valued neutrosophic binary relation. A typical C-style description follows:

```
typedef struct vtif {
    double t;
```

```
    double i;
    double f;
    } TIF;
TIF R[10][15], S[15][5], RS[10][5];
```

With these considerations, the following rules can be used:

$$RS[u][w].t = \sup\left\{\min(R[u][v].t), S[v][w].t\right| \text{ for all } v \in V\right\},$$

$$RS[u][w].i = \sup\left\{\min(R[u][v].i), S[v][w].i\right| \text{ for all } v \in V\right\},$$

and

$$RS[u][w].f = \sup\left\{\min(R[u][v].f), S[v][w].f\right| \text{ for all } v \in V\right\}.$$

Also, a list of TIF elements can be used. A linked list describing the relation R can be defined as:

```
typedef struct vel {
    U           u;
    V           v;
    TIF         NC; /* the neutrosophic components */
    struct vel * next;
    } VNODE;
VNODE *R, *S, *RS;
```

An interval valued neutrosophic binary relation is based on `INTERVAL` data structure:

```
typedef struct interval {
    double left_margin;
    double right_margin;
    } INTERVAL;
typedef struct vtif {
    INTERVAL t;
    INTERVAL i;
    INTERVAL f;
    } VTIF;
```

When defined as list, an interval valued neutrosophic binary relation could be defined as:

```
typedef struct iel {
    U           u;
    V           v;
    VTIF        NC; /* the neutrosophic components */
    struct iel * next;
    } INODE;
    INODE *R, *S, *RS;
```

In the general case the neutrosophic components can be unions of values and interval values. Any single value can be viewed as interval valued with identical left and right borders. In this case a list of intervals can be defined:

```
typedef struct ivlist {
unsigned int  id; /* interval identification number */
    INTERVAL      iv; /* iv is an INTERVAL data structure */
    struct ivlist * next;
} IVLIST;
typedef struct ivtif {
    IVLIST *t;
    IVLIST *i;
    IVLIST *f;
    } IVTIF;
typedef struct ivel {
    U           u;
    V           v;
    IVTIF        NC; /* the neutrosophic components */
    struct ivel * next;
} IVNODE;
IVNODE *R, *S, *RS;
```

Once the appropriate data structures were designed, the implementation of methods based on algorithms described above is possible. The presented framework is general enough to permit implementations based on object oriented paradigm.

Finally, let us consider the natural extension of neutrosophic relations which uses n crisp sets (domains) $D_1, D_2, \ldots, D_n$. A n-components record r with fields $d_1, d_2, \ldots, d_n$ having attached a TIF element t_r is a neutrosophic record in a neutrosophic database. If $I(r) = 0$ for all r, then the neutrosophic relational model described by [33] is obtained, and if $T(r) + F(r) = 1$ for every r, the relation is a *total neutrosophic relation*; $T(r)$ is called *belief* index, and $F(r)$ is called *doubt* index in [33]. For database oriented applications both *normalization* rules and *constraints* over the neutrosophic components are required. Various type of constraints and approaches were already considered in [1, 2, 3].

4. Conclusion

Neutrosophic thinking is used in many fields of scientific research. This paper reviewed some developments in order to identify the principles of neutrosophic computing useful to software implementation for a large plethora of applications.

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